MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2016/2017

BSP2014 – INTRODUCTION TO APPLIED PROBABILITY AND STOCHASTIC PROCESSES

(All sections / Groups)

3 MARCH 2017
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of **FIVE (5)** printed pages inclusive of the formulae sheet.

2. Answer **ALL** four questions in the answer booklet provided.

3. Students are allowed to use authorized calculators by lecturer only.

4. Marks are shown at the end of each question.

Question 1 [25 marks]

(a) The random variable X has the pdf of:

$$f(x)=\begin{cases} c \quad , & 0 \le x \le 1 \\ cx^{-2} \quad , & x \ge 1 \end{cases}$$

(i) Determine the value of c. [4 marks]
(ii) Find P (X > 0.5). [3 marks]
(iii) Find P(X > 2). [2 marks]

(b) The random variable X is uniform on [10, 250],
(i) Find the probability density function for random variable X. [1 mark]
(ii) Determine the moment generating function for X. [3 marks]
(iii) Find $E(3X^2-2X)$. [8 marks]
(iv) Find Var (2X + 1). [4 marks]

Question 2 [25 marks]

(a) The joint pmf of random variable X and Y is given by

$$f(x,y)=\frac{1}{30}x^2y \quad ; \qquad x=1,2 \quad ; \quad y=1,2,3$$

(i) Determine the marginal pmf for X and Y respectively. [6 marks]
(ii) Calculate E(XY). [6 marks]
(iii) Calculate Cov (X,Y). [6 marks]
(iv) Is X and Y independent? Explain. [1 mark]

(b) Astro would like to know about their market in Klang Valley and recent report shows that 65% of households in Klang Valley installed Astro at home.
(i) What is the probability of 6 households checked to get the first house installed with Astro? [2 marks]
(ii) What is the probability of at least 3 households checked to get the first house without Astro? [4 marks]

Continued...

Question 3 [25 marks]

(a) Buses arrive at the main bus stop within UPM to take students to The Mines shopping mall with an average of 5 buses per hour. Christ just missed the last bus.
   (i) What is the probability that he need to wait more than 20 minutes for the next bus to arrive? [5 marks]
   (ii) What is the probability that the bus arrive within 10 minutes. [5 marks]
   (iii) What is the expected inter-arrival time of the buses? [2 marks]

(b) A random walk starts from the origin. The chances for the walker to move to the right are triple the chances he moves to the left.
   (i) Find the probability that the walker is at the origin after 6 steps. [6 marks]
   (ii) Find the probability that the walker is at position -3 after 9 steps. [4 marks]
   (iii) Find the mean of the position after 20 moves. [3 marks]

Question 4 [25 marks]

Consider a Markov chain with state $\{1, 2\}$ and transition probability matrix

$$P = \begin{bmatrix} 0.75 & 0.25 \\ 0.5 & 0.5 \end{bmatrix}$$

(a) Draw the transition diagram. [3 marks]
(b) Determine for every state whether it is absorbing. [4 marks]
(c) Determine for every state whether it is transient or persistent. [12 marks]
(d) Determine the stationary distribution. [6 marks]

End of Page

FORMULAE

A. PROBABILITY DISTRIBUTION

Bernoulli Probability Distribution

$$P(X = x) = p^x q^{1-x} \quad \text{for } x = 0, 1$$

Binomial Probability Distribution

$$P(X = x) = {}^nC_x \, p^x q^{n-x} \quad \text{for } x = 0, 1, \ldots, n$$

Poisson Probability Distribution

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!} \quad \text{for } x = 0, 1, 2, \ldots$$

Geometric Probability Distribution

$$P(X = x) = pq^{x-1} \quad \text{for } x = 1, 2, \ldots$$

Uniform Probability Distribution

$$f(x) = \frac{1}{b-a} \quad \text{for } a < x < b$$

Exponential Probability Distribution

$$f(x) = \lambda e^{-\lambda x} \quad \text{for } x > 0$$

Normal Probability Distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad \text{for } -\infty < x < \infty$$

Gamma Probability Distribution

$$f(x) = \frac{x^{\alpha-1} e^{-x/\beta}}{\beta^\alpha \Gamma(\alpha)} \quad \text{for } x \geq 0$$

where $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$ for $\alpha \geq 0$

with $\Gamma(\alpha) = (\alpha - 1)\,\Gamma(\alpha - 1)$

with $\Gamma(n) = (n - 1)!$ when n is a positive integer.

Chi-Square Probability Distribution

$$f(x) = \frac{x^{\left(\frac{n}{2}-1\right)} e^{-x/2}}{2^{\frac{n}{2}}\Gamma\left(\frac{n}{2}\right)} \quad \text{for } x \geq 0$$

B. MOMENT

k^{th} Moment about the origin of X:

$$\mu'_k = E[X^k]$$

k^{th} Moment about the mean of X:

$$\mu_k = E[X-\mu]^k$$

Moment Generating Function for X:

$$M_X(t) = E\left(e^{tx}\right)$$

Derivative of Moment Generating Function for X:

$$M_X^{(r)}(t) = E\left(X^r e^{tx}\right)$$

C. STOCHASTIC PROCESS

Simple Random Walk:

$$P(X_n = m) = \binom{n}{\frac{n+m}{2}} p^{(\frac{n+m}{2})} q^{(\frac{n-m}{2})}$$ where m and n are both even or both odd.

$E(X_n) = n(p-q)$; $Var(X_n) = 4npq$

Gambler's Ruin Problem

$$P_a = \begin{cases} \dfrac{(\frac{q}{p})^a - (\frac{q}{p})^c}{1-(\frac{q}{p})^c}, & p \neq q \\ 1-\dfrac{a}{c}, & p = q \end{cases} \quad ; \quad P_b + P_a = 1$$

$$D_a = a(c-a) \quad \text{if} \quad p = q$$

$$D_a = \frac{1}{q-p}\left\{a - c\left[\frac{1-\left(\frac{q}{p}\right)^a}{1-\left(\frac{q}{p}\right)^c}\right]\right\} \quad \text{if} \quad p \neq q$$

D. POISSON PROCESS

Pmf for N(t) : $$P\{N(t)=n\}=\frac{e^{-\lambda t}(\lambda t)^n}{n!}$$
Pdf for T_n: $$f(t)=\lambda e^{-\lambda t}$$
Pdf for S_n : $$f(t)=\frac{\lambda e^{-\lambda t}(\lambda t)^{n-1}}{(n-1)!}$$